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The Lewis factor and its influence on the performance prediction of wet-cooling towers

Johannes C. Kloppers [a,*], Detlev G. Kröger [b]

[a] *Sasol Technology (Pty) Ltd, Private Bag X1034, Secunda 2302, South Africa*
[b] *Department of Mechanical Engineering, University of Stellenbosch, Stellenbosch 7600, South Africa*

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Abstract

The effect of the Lewis factor, or Lewis relation, on the performance prediction of natural draft and mechanical draft wet-cooling towers is investigated. The Lewis factor relates the relative rates of heat and mass transfer in wet-cooling towers. The history and development of the Lewis factor and its application in wet-cooling tower heat and mass transfer analyses are discussed. The relation of the Lewis factor to the Lewis number is also investigated. The influence of the Lewis factor on the prediction of wet-cooling tower performance is subsequently investigated. The Poppe heat and mass transfer analysis of evaporative cooling are considered as the Lewis factor can be explicitly specified. It is found that if the same definition or value of the Lewis factor is employed in the fill test analysis and in the subsequent cooling tower performance analysis, the water outlet temperature will be accurately predicted. The amount of water that evaporates, however, is a function of the actual value of the Lewis factor. If the inlet ambient air temperature is relatively high, the influence of the Lewis factor, on tower performance diminishes. It is very important, in the view of the Lewis factor that any cooling tower fill test be conducted under conditions that are as close as possible to the conditions specified for cooling tower operating conditions.

Keywords: Lewis factor; Lewis number; Wet-cooling tower; Evaporation; Poppe

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1. Introduction

The Lewis factor, Le_f, appears in the governing equations of the heat and mass transfer processes (evaporative cooling) in a wet-cooling tower according to Merkel [1] and Poppe and Rögener [2]. Merkel [1] assumed that the Lewis factor is equal to 1 to simplify the governing equations while Poppe and Rögener [2] used the equation of Bosnjakovic [3] to express the Lewis factor in their more rigorous approach. This approach is commonly known as the Poppe method and will be referred as such in this paper. The analysis of Poppe [2] is employed in the current investigation as the value of the Lewis factor can be explicitly specified. The influence of the Lewis factor on wet-cooling tower performance can therefore be critically evaluated under a wide range of ambient conditions.

There is a common misconception among researchers who refer to the Lewis number, Le, as the Lewis factor, Le_f. The relation between the Lewis number and the Lewis factor is explained.

2. Lewis number

The derivation and significance of the Lewis number, Le, is explained by its analogy to the derivation of the Prandtl, Pr, and Schmidt, Sc, numbers.

The rate equation for momentum transfer is given by Newton's law of viscosity, i.e.,

$$\frac{F}{\mathrm{A}} = -\mu \frac{\partial u}{\partial y} = -\nu \frac{\partial (\rho u)}{\partial y} \tag{1}$$

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[*] Corresponding author. Tel.: + 27 (0)17 6192739; fax: + 27 (0)11 5220599.

E-mail addresses: chris.kloppers@sasol.com (J.C. Kloppers), dgk@sun.ac.za (D.G. Kröger).

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Nomenclature

Symbol	Description	Unit
A	area	m^2
C	coefficient	
c_i	concentration	$kg \cdot m^{-3}$
c_p	specific heat at constant pressure	$J \cdot kg^{-1} \cdot K^{-1}$
D	diffusion coefficient	$m^2 \cdot s^{-1}$
d	molecular weight ratio $= 0.622$	
F	force	N
h	heat transfer coefficient	$W \cdot m^{-2} \cdot K^{-1}$
h_m	mass transfer coefficient	$kg \cdot m^{-2} \cdot s^{-1}$
k	thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
L	length, m, or length scale	
$\dot{M}$	mass flow rate	$kg \cdot s^{-1}$
Q	heat transfer rate	W
T	time scale	
u	velocity	$m \cdot s^{-1}$
w	humidity ratio	(kg water vapor)·(kg dry air)$^{-1}$
y	coordinate	

Greek symbols

Symbol	Description	Unit
α	thermal diffusivity, $k/\rho c_p$	$m^2 \cdot s^{-1}$
μ	dynamic viscosity	$kg \cdot m^{-1} \cdot s^{-1}$
ν	kinematic viscosity	$m^2 \cdot s^{-1}$
ρ	density	$kg \cdot m^{-3}$

Dimensionless numbers

Symbol	Description
Le	Lewis number, $k/(\rho c_p D)$, or Sc/Pr, or α/D
Le_f	Lewis factor, $h/(c_p h_m)$
Nu	Nusselt number, hL/k
Pr	Prandtl number, $c_p \mu/k$
Re	Reynolds number, $\rho u L/\mu$
Sc	Schmidt number, $\mu/(\rho D)$
Sh	Sherwood number, $h_m L/D$
St	Stanton number, $h/(\rho u c_p)$, or $Nu/(Re\,Pr)$
St_m	mass transfer Stanton number, $h_m/(\rho \nu)$ or $Sh/(Re\,Sc)$

Subscripts

Symbol	Description
a	air
i	inlet
m	mass transfer, or mean
o	outlet
s	saturation
w	water

The rate equation for heat or energy transfer is given by Fourier's law of heat conduction,

$$\frac{Q}{A} = -k\frac{\partial T}{\partial y} = -\alpha\frac{\partial(\rho c_p T)}{\partial y} \tag{2}$$

The rate equation for mass transfer is given by Fick's law of diffusion, i.e.,

$$\frac{\dot{M}}{A} = -D\frac{\partial c_i}{\partial y} \tag{3}$$

The diffusivities ν, α and D in Eqs. (1)–(3) have dimensions of $[L^2/T]$, where L and T refer to the length and time scales respectively. Any ratio of two of these coefficients will result in a dimensionless number. In systems undergoing simultaneous convective heat and momentum transfer, the ratio of ν to α would be of importance and is defined as the Prandtl number, i.e.,

$$Pr = \frac{\nu}{\alpha} = \frac{c_p \mu}{k} \tag{4}$$

In processes involving simultaneous momentum and mass transfer the Schmidt number is defined as the ratio of ν to D, i.e.,

$$Sc = \frac{\nu}{D} \tag{5}$$

In processes involving simultaneous convective heat and mass transfer, the ratio of α to D is defined as the Lewis number, i.e.,

$$Le = \frac{\alpha}{D} = \frac{k}{\rho c_p D} = \frac{Sc}{Pr} \tag{6}$$

From Eq. (6) it can be seen that the Lewis number is equal to the ratio of the Schmidt to the Prandtl number and is relevant to simultaneous convective heat and mass transfer. The relative rate of growth of the thermal and concentration boundary layers are determined by the Lewis number. The temperature and concentration profiles will coincide when $Le = 1$.

3. Lewis factor

In addition to the Lewis number, *Le*, the Lewis factor, or Lewis relation, Le_f, can be defined: it gives an indication of the relative rates of heat and mass transfer in an evaporative process. In some of the literature encountered there seems to be confusion about the definitions of these dimensionless numbers and the Lewis factor is often incorrectly referred to as the Lewis number.

The Lewis factor, Le_f, is equal to the ratio of the heat transfer Stanton number, *St*, to the mass transfer Stanton number, St_m where

$$St = \frac{Nu}{Re\,Pr} = \frac{h}{\rho u c_p} \tag{7}$$

$$St_m = \frac{Sh}{Re\,Sc} = \frac{h_m}{\rho u} \tag{8}$$

where *Nu* is the Nusselt number, or dimensionless heat flux, and *Sh* is the Sherwood number, or dimensionless mass flux.

The Lewis factor can be obtained by dividing Eq. (7) by Eq. (8), i.e.,

$$Le_f = \frac{St}{St_m} = \frac{h}{\rho u c_p}\frac{\rho u}{h_m} = \frac{h}{c_p h_m} \tag{9}$$

Lewis [4] tried to prove analytically that $Le_f = 1$ for gas/liquid systems. In a later article [5] he stated that the relation, $Le_f = 1$, holds approximately for air/water mixtures but not for all liquid–gas mixtures. Although the proof given by Lewis was incorrect [6], the ratio $h/c_p h_m$ is today known as the Lewis factor.

From the Chilton–Colburn analogy power law relations it follows that [17],

$$St = C\,Re^{-1/2}Pr^{-2/3} = \frac{h}{\rho u c_p} \tag{10}$$

$$St_m = C\,Re^{-1/2}Sc^{-2/3} = \frac{h_m}{\rho u} \tag{11}$$

It then follows that

$$Le_f = \frac{St}{St_m} = \left(\frac{Pr}{Sc}\right)^{-2/3} = Le^{2/3} \tag{12}$$

Bourillot [7] states that the Lewis number is not constant and is tied to the nature of the vapor–gas mixture. It also depends on the nature of the boundary layer near the exchange surfaces and the thermodynamic state of the mixture [7,8]. Bosnjakovic [3] pointed out that the mass transfer is not proportional to the humidity potential, $(w_{sw} - w)$. A corrector term, $F(\xi)$, is applied to Eq. (12) and the expression for Le_f in the Bosnjakovic form is obtained.

$$Le_f = Le^{2/3}\frac{1}{F(\xi)} \tag{13}$$

where

$$F(\xi) = \frac{\ln\xi}{\xi - 1} \quad \text{and} \quad \xi = \frac{w_{sw} + d}{w + d}$$

where $d =$ Molecular weight of water/Molecular weight of air $= 0.622$.

Poppe and Rögener [2] cited that the Lewis factor, Le_f, is according to the Bosnjakovic form,

$$\begin{aligned} Le_f &= 0.865^{2/3}\frac{1}{F(\xi)} \\ &= 0.865^{2/3}\left[\left(\frac{w_{sw} + d}{w + d} - 1\right)\Big/\ln\left(\frac{w_{sw} + d}{w + d}\right)\right] \end{aligned} \tag{14}$$

where the Lewis number, *Le*, is assumed constant at 0.865. Bourillot [9] and Grange [8] state that the Lewis factor, Le_f, for a wet-cooling tower, using Eq. (14), is approximately 0.92.

In his classical work on evaporation Merkel [1] assumed that $Le_f = 1$. Häszler [10] reports that the assumption of Merkel is not correct and that Lewis factors are in the range from 0.5 to 1.3. An analysis of both splash and film packings by Feltzin and Benton [11] indicates that for counterflow towers a Lewis factor of 1.25 is more appropriate. Sutherland [12] used a Lewis factor of 0.9 in his “accurate” tower analysis.

Häszler [10] states that when the humidity potential $(w_{sw} - w)$ is large, Eq. (12) is not valid any more.

4. Influence of Lewis factor on cooling tower performance evaluation

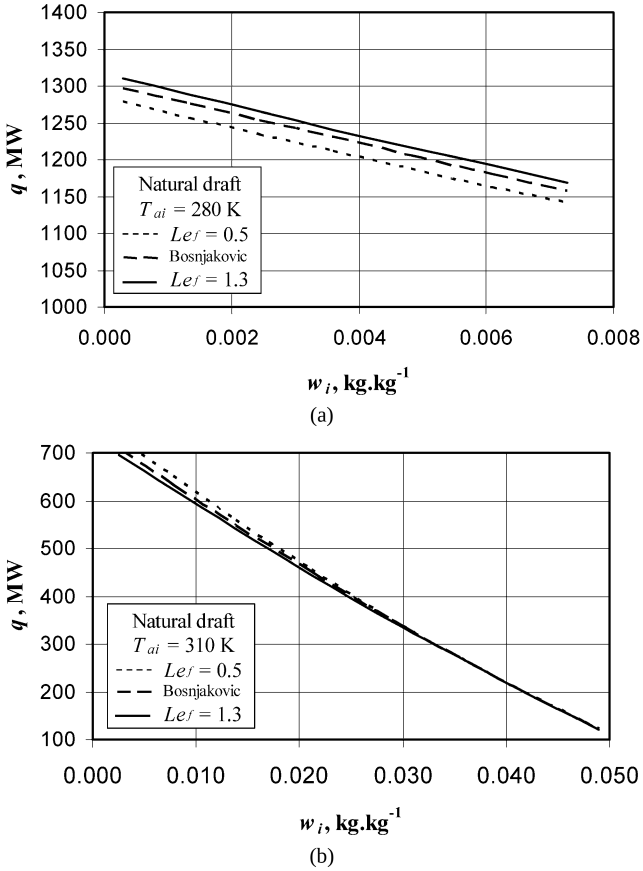
Three different specifications of the Lewis factor are employed in this investigation to determine the effect of the Lewis factor on the performance prediction of natural draft wet-cooling towers. The trends that are observed are applicable to mechanical draft towers as well. Eq. (13) is employed, which predicts Lewis factors of approximately 0.92, as well as the limiting values cited by Häszler [10] of 0.5 and 1.3. The same definition of the Lewis factor is employed in the fill performance analysis and the subsequent cooling tower performance analysis.

The method employed in this investigation to solve the governing heat and mass transfer equations for counterflow cooling towers, according to the Poppe [2] method, is described in detail in Kloppers and Kröger [13]. The application of these equations to cooling tower performance evaluation can be found in Kloppers and Kröger [14]. Also refer to Kloppers and Kröger [15] where the solution method of the equations for crossflow cooling towers is discussed. As already mentioned, the Poppe [2] method is employed to study the influence of the Lewis factor on cooling tower performance as the Lewis factor can be explicitly specified. Details of the natural draft cooling tower employed in this analysis are given in Kröger [16]. Wet-cooling tower performance prediction software, developed by the authors, is employed in this investigation.

The differences between the results according to the different values of the Lewis factor are investigated at various operating conditions. Ambient air temperatures of 280, 290, 300 and 310 K are considered. The humidity of the air is varied from completely dry to saturated conditions at each of the four selected temperatures. The effect of the Lewis factor on cooling tower performance can therefore be determined over a wide range of atmospheric conditions. The graphical results that follow are for ambient temperatures of 280 and 310 K.

4.1. Heat rejection rate

The heat rejection rates for the different specifications of the Lewis factor for dry to saturated atmospheric conditions at 280 and 310 K, can be seen in Figs. 1(a) and 2(b), respectively. The higher the Lewis factor, the more heat is rejected. In the natural draft cooling tower at an ambient temperature of 280 K the differences in heat rejection rates, between the analyses with Lewis factors of 0.5 and 1.3, are approximately 2.4%. The difference is 0.8% at 290 K and



(a)

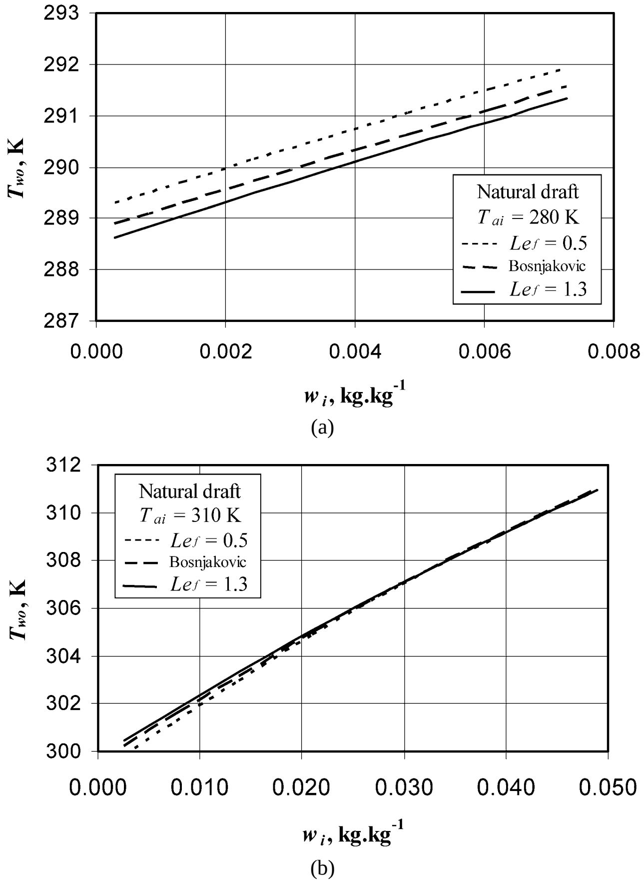
(b)

Fig. 1. The difference between the heat rejection rates predicted by three different values of Lewis factors for varying atmospheric humidities at 280 and 310 K.

approximately zero at 300 K. At 310 K in very dry conditions, the difference is almost 5% where the heat rejected, due to the smaller Lewis factor, is more than that predicted by the higher Lewis factor. When the inlet ambient air is relatively hot and humid there is virtually no difference between the results as can be seen in Fig. 1(b).

4.2. Water outlet temperature

Because more heat is rejected at higher Lewis factors, the corresponding water outlet temperature is lower. The discrepancy between the water outlet temperatures, by applying Lewis factors of 0.5 and 1.3, respectively, is approximately 0.6 K at an ambient temperature of 280 K for all humidities and at 310 K for very low humidity. This discrepancy is practically zero at 300 K. When the inlet ambient air is relatively hot and humid there is again virtually no difference between the results of the different definitions of the Lewis factor for the water outlet temperatures as shown in Fig. 2(b).



(a)

(b)

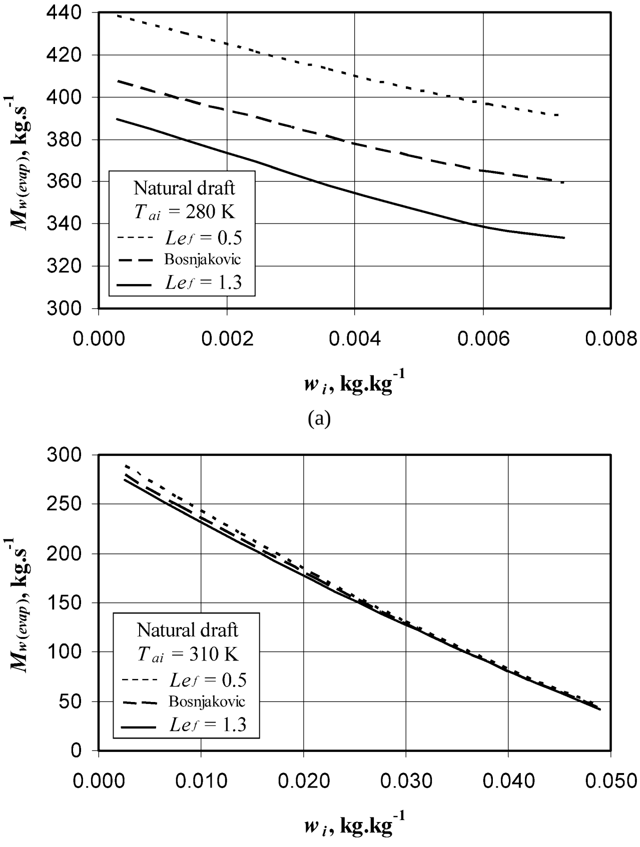
Fig. 2. The difference between the water outlet temperatures predicted by three different values of Lewis factors for varying atmospheric humidities at 280 and 310 K.

When the transfer coefficient, or Merkel number, is determined during a cooling tower fill performance test, the water outlet temperature is known. In the subsequent cooling tower performance test, the transfer coefficient is known and the water outlet temperature is generally unknown. The outlet water temperature can therefore be determined from the known transfer coefficient. If the same method of analysis is employed, at the same ambient conditions, in the fill performance test and the subsequent cooling tower performance test, the water outlet temperature must be the same regardless of the method of analysis (i.e., Merkel [1] or Poppe [2]) that are employed. But it can be seen from Fig. 2 that the water outlet temperature is not the same, even though the same analysis and definitions are employed in the fill performance test and the subsequent cooling tower performance analysis. It is therefore imperative that the cooling tower performance be evaluated at the same ambient conditions under which the performance of the fill was tested.

4.3. Water evaporation rate

As can be seen from Fig. 3, the water evaporation rate is higher when applying smaller Lewis factors than with higher ones. Thus, the air becomes saturated more quickly with lower Lewis factors. The discrepancy between the water evaporation rates in cooling towers with Lewis factors of 0.5 and 1.3, is approximately 15% at 280 K and reduces to 6% at 310 K.

Grange [8] shows in a comparative study that the Merkel [1] method tends to underestimate the amount of water that



(a)

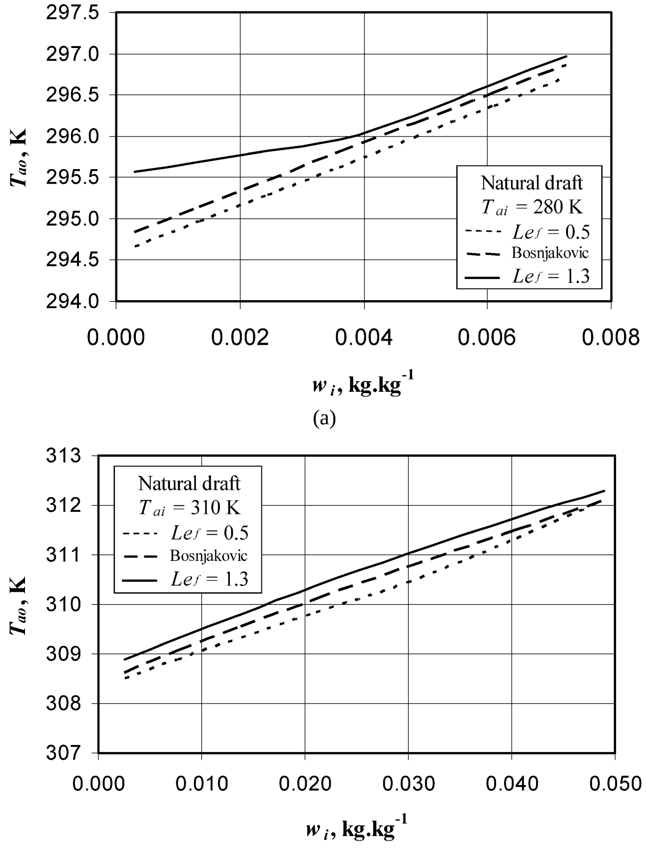
(b)

Fig. 3. The difference between the water evaporation rates predicted by three different values of Lewis factors for varying atmospheric humidities at 280 and 310 K.

evaporates when compared to the Poppe [2] analysis, but that the discrepancy decreases with increasing ambient temperatures. The results of Grange [8] were verified by the authors. It is therefore clear that the influence of the assumptions and definitions of the Merkel and Poppe analyses regarding the Lewis factor, on the results of cooling tower performance, diminishes when the inlet ambient air is relatively hot and humid.

4.4. Air outlet temperature

From Fig. 4 it can be seen that the higher the Lewis factor, the higher the air outlet temperature. It can be seen from Fig. 4(a) that the trends for the predicted air outlet temperatures, according to the different Lewis factors, are not the same. This is because the outlet air is unsaturated for the $Le_f = 1.3$ case below a humidity ratio of approximately 0.0035. For the other cases in Fig. 4(a) and all the cases in Fig. 4(b), the outlet air is supersaturated with water vapor. As already mentioned, the outlet air becomes saturated more quickly for lower Lewis factors. Refer to Kloppers and Kröger [14] for a detailed discussion why the trends differ between unsaturated and supersaturated air.



(a)

(b)

Fig. 4. The difference between the air outlet temperatures predicted by three different values of Lewis factors for varying atmospheric humidities at 280 and 310 K.

4.5. Discussion on the consistent application of the Lewis factor

The Lewis factor is consistently applied when the same definition or equation of the Lewis factor is employed in the fill performance analysis and in the subsequent cooling tower performance analysis. The Lewis factor has little influence on the water outlet temperature and the heat rejected from the cooling tower in very humid ambient air. In dry conditions, at all ambient temperatures considered, the differences between the results of the different Lewis factors can be quite significant. The rate of water evaporation is strongly dependent on the Lewis factor for both the natural draft and mechanical draft towers. This is because the Lewis factor is an indication of the relative rates of heat and mass transfer in an evaporative process. The Lewis factor can therefore be tuned to represent the physically measured evaporation rates and outlet air temperatures more closely in fill performance analyses. It is therefore important to perform the fill performance tests in conditions that closely represent actual operational conditions, especially if the cooling tower is operated at a very low ambient humidity.

If the fill performance test data is insufficient to accurately predicts the Lewis factor of a particular fill, it is recommended that the equation of Bosnjakovic be used as the numerical value is approximately 0.92, which is approximately the mean between the limiting values of 0.5 and 1.3 given by Häszler [10].

4.6. The inconsistent application of the Lewis factor

The analyses of the natural draft cooling towers are repeated with an inconsistent application of the Lewis factor specification, i.e. the equation of Bosnjakovic [3] is used in the fill performance evaluation, while Lewis factors of 0.5 and 1.3 are used in the cooling tower performance evaluation.

The inconsistent application of the Lewis factor results in larger discrepancies than is the case with the consistent application of the Lewis factor. The discrepancy between the heat rejection rate is approximately 8% at an ambient temperature of 280 K. The discrepancy is only 2.4% where the Lewis factors are applied consistently. The discrepancy reduces at higher ambient temperatures to approximately 2% at 310 K. This is consistent with the conclusion reached previously, that the influence of the Lewis factor diminishes at higher ambient temperatures. The discrepancy in the water outlet temperature and air outlet temperature for the natural draft cooling tower, for the inconsistent analysis of the Lewis factor, is larger than the consistent application.

5. Conclusion

Exactly the same definition of the Lewis factor must be employed in the fill performance analysis and in the subsequent cooling tower performance analysis. The fill performance test must be conducted at, or as close as possible, to conditions specified for operation of the cooling tower for which it is intended. The influence of the Lewis factor, on the performance evaluation of wet-cooling towers, diminishes when the inlet ambient air is relatively hot and humid. For increasing Lewis factors, the heat rejection rate increases, the water outlet temperature decreases and the water evaporation rate decreases.

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